CONFINEMENT PHASE TRANSITION IN GLUODYNAMICS VIA VARIABLES AT INFINITY

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Abstract

The general mechanism of confinement phase transition is clarified in 3-d Fock–Schwinger gauge at finite temperature for SU(2) and SU(3) gluodynamics. The variables at infinity initially introduced in the algebraic QFT play the essential role for this phase transition. In more conventional terms the arising variable at infinity σ_∞ is the expectation value of the gauge field temporal component A_0. So the considered mechanism is related with the problem of A_0-condensation.

It was early realized that the Yang–Mils theory should possess the property of confinement. However even nowadays this problem does not appear as one being firmly established. Various approaches were tried in hope to achieve comprehension of this phenomenon [1], yet with just mild success. May be the reason is inefficiency of perturbation expansions as long as truly collective effects are concerned. The main aim of this report is to outline a reliable theoretical framework for the effect under consideration. We present here some preliminary results of our nonperturbative approach that is plausibly capable to shed light on the underlain physics.

In such treatment removal of superfluous (unphysical) degrees of freedom would be highly desirable. Therefore one has to fix a suitable physical gauge to work in. We choose the 3-dimensional Fock–Schwinger gauge [2-4]

$$\hat{\mathbf{x}}\,\mathbf{A}(t,\mathbf{x}) = 0\,, \qquad \hat{\mathbf{x}} = \frac{\mathbf{x}}{x}\,, \quad x = |\,\mathbf{x}\,|\,, \tag{1}$$

where the $\mathbf{x}$-transversal components of electric field strengths $\mathbf{E}_\perp$ and gauge fields $\mathbf{A}_\perp$ are the canonical variables of YM theory obeying the CCR. The Gauss law constraint $\nabla\mathbf{E} = 0$ possesses in FS gauge the explicit simple solution

$$\mathbf{E} = \mathbf{E}_\perp + \hat{\mathbf{x}}\, E_\parallel\,, \qquad E_\parallel(\mathbf{x}) = -\,\frac{1}{x^2}\int_0^x y^2\, dy\, \nabla\mathbf{E}_\perp(y\hat{\mathbf{x}})\,. \tag{2}$$

If one plugs the latter into the YM Hamiltonian this yields no more than quartic expression versus canonical variables. Let us indicate that we sacrifice manifest Poincaré covariance and locality of the Hamiltonian to get these simplifications.

Next we consider the YM partition function at inverse temperature β written as path integral over fields periodic in "time" t

$$Z = \int \mathcal{D}\mathbf{A}_\perp\, \mathcal{D}\mathbf{E}_\perp\, \exp\Big[\int_0^\beta dt \int d\mathbf{x}\, f_R^2\,(\mathbf{x})(i\,\dot{\mathbf{A}}_\perp\mathbf{E}_\perp - \frac{1}{2}\mathbf{E}^2 - \frac{1}{2}\mathbf{B}^2)\Big]\,, \tag{3}$$

where $\mathbf{B}$ denotes the magnetic field strength. The cut–off function f_R with the properties

$$f_R(\mathbf{x}) = \begin{cases} 1, & |\,\mathbf{x}\,| \le R' \qquad R' = (1-\varepsilon)R \\ 0, & |\,\mathbf{x}\,| \ge R'' \qquad R'' = (1+\varepsilon)R \end{cases} \tag{4}$$

was introduced in (3) to prevent infrared singularities. To proceed further we exploit the simple observation that only the longitudinal parts of field strengths contain nonlocal as well as nonabelian terms. Namely we make the integrand of (3) gaussian in $\mathbf{A}_\perp$ and $\mathbf{E}_\perp$ by introducing new "scalar" fields via

$$e^{-\frac{1}{2}\int dx\, f_R^2\,(E_\parallel^2 + B_\parallel^2)} = \int \mathcal{D}\lambda\,\mathcal{D}\nu\, e^{\int dx\,[-\frac{1}{2}(\lambda^2+\nu^2) + i\, f_R\,(\lambda\, E_\parallel + \nu\, B_\parallel)]}. \tag{5}$$

Further step is in excluding of nonlocality

$$\int d\mathbf{x}\, f_R(x)\,\lambda(\mathbf{x})\, E_\parallel(\mathbf{x}) = \int d\mathbf{x}\,\nabla\mathbf{E}_\perp(\mathbf{x})(f_R(x)\sigma(\mathbf{x}) + \sigma_R(\mathbf{x}))\,, \tag{6}$$

where the new variables are

$$\sigma(\mathbf{x}) = \int_x^\infty dy\,\lambda(y\hat{\mathbf{x}})\,, \qquad \sigma_R(\mathbf{x}) = \int_x^\infty dy\, f_R^{'}(y)\,\sigma(y\hat{\mathbf{x}})\,. \tag{7}$$

In terms of ν, σ and σ_R the theory looks like effectively localized. Note however that $\sigma_R(\mathbf{x})$ contains purely delocalized contributions after the infrared cut–off removing ($R \to \infty$) because supp $f'_R(x) = [R', R'']$. Accordingly it fails to possess a well–defined limit unless special conditions on the space of states have been imposed. There lies a deep algebraic theory behind it developed mainly by G. Morchio and F. Strocchi [7-9]. From this point of view σ_R tends to what is called a *variable at infinity* $\sigma_R(\mathbf{x}) \stackrel{R\to\infty}{\longrightarrow} \sigma_\infty(\hat{\mathbf{x}})$. At present stage σ_∞ may be looked at as some kind of order parameter appearing in the YM theory, and our next task is to find out it's equilibrium value.

Performing integrations over $\mathbf{A}_\perp,\ \mathbf{E}_\perp$ in (3) (see [10] for more detail) we get

$$Z[\beta,\sigma_\infty] = \int \mathcal{D}\,\sigma\,\mathcal{D}\,\nu\ \exp\left(-W[\sigma+\sigma_\infty,\nu]\right),$$

$$W[\sigma,\nu] = \tfrac{1}{2}\nu^2 - \tfrac{1}{2}\sigma\,\Delta\,\sigma + \tfrac{1}{2}K_-\bullet C_+^{-1}\bullet K_+ + \tfrac{1}{2}K_+\bullet C_-^{-1}\bullet K_- + \tfrac{1}{2}\mathrm{tr}\,\log\,C_+C_- \quad (8)$$

$$C_\pm = -\Delta_x - \nabla_t^2 \pm D\,,\quad K_\pm = \partial_\pm\,\nu \pm \nabla_t\,\partial_\pm\,\sigma\,, \quad (9)$$

$$\nabla_t^{ab} = \delta^{ab}\,\partial_t - gt^{abc}\,\sigma^c\,,\qquad D^{ab} = gt^{abc}\,\nu^c\,. \quad (10)$$

In (8) the dot denotes integration over t and $\mathbf{x}$ together with summation over space and colour indices, Δ_x is the radial part of the Laplacian. Projected derivatives are defined by means of

$$\partial^i_\pm = \Pi^{ij}_\pm\,\partial_j\,,\qquad \Pi^{ij}_\pm = \frac{1}{2}(\delta^{ij} - \hat{x}^i\,\hat{x}^j \pm i\,\varepsilon^{ijk}\,\hat{x}^k)\,. \quad (11)$$

The saddle point method at the "classical" level requires to look for minima of the effective action $W[\sigma,\nu]$. The simplest possible *Ansatz* of constant σ and ν is used that is, however, rather natural from physical viewpoint. Denoting[1] $v^a = i\frac{\beta g}{2\pi}\nu^a$, $s^a = \frac{\beta g}{2\pi}\sigma^a$ we present the effective potential (= free energy density) in the form

$$\mathcal{F} = \frac{8\pi^2}{\beta^2\,g^2}\,\chi_0\,F[s^a,v^a]\,, \quad (12)$$

clearly emphasizing its nonperturbative character. Here

$$F[s^a,v^a] = -a(v^a)^2 + \mathcal{U}[s^a,v^a]\,,\qquad \mathcal{U}[s^a,v^a] = \pi^{-1}\sum_{n=-\infty}^{\infty}\int_0^\infty dq\,\log\,\det\mathcal{C}\,, \quad (13)$$

$$\mathcal{C}^{ab}[s^a,v^a] = \delta^{ab}(q^2+n^2) + i\,t^{abc}(2n\,s^c + v^c) + t^{abc}\,t^{bcd}\,s^cs^e \quad (14)$$

[1] both s^a and v^a are real

with

$$a = \frac{\pi^2}{\beta^2\,\chi_0}\,.$$

In (12), χ_0 is a certain combination of infrared and ultraviolet regulators with dimension of $(mass)^2$. The consistent theory would somehow relate it to confinement radius and UV normalization point, but this theory is still far beyond of our present knowledge. Nevertheless it may be shown [11, 12] that χ_0 is nothing but the string tension of SU(2) gluodynamics!

For the SU(2) group, explicit analysis [10] of function $\mathcal{U}[s^a, v^a]$ shows that the vectors s^a, v^a must be collinear, otherwise the imaginary part of F is nonvanishing. Then $\mathcal{U}$ is periodic versus s, and for $0 \le s \le 1$

$$\mathcal{U}[s,v] = -\frac{1}{2}(1-2s)^2 + \sum_{k=-\infty}^{\infty}\left(\sqrt{(k+s)^2+v} + \sqrt{(k+s)^2-v} - 2\,|\,k+s\,|\right). \tag{15}$$

The real part of F is a rather complicated function. The available positions of its minima are $s=0$, $u=n$ and $s=\frac{1}{2}$, $u=n+\frac{1}{2}$ with $n \in \mathbf{Z}$, and $u=\sqrt{|\,v\,|}$. In any case, however, just two of them with $n=0$ are stable belonging to the region where $\mathrm{Im}\,F=0$. The second minimum is the deeper one, but it emerges only below the critical temperature

$$T_c = \sqrt{a_c\,\chi_0}/\pi\,,$$

$$a_c = 2\sqrt{2} - 8\sum_{k=0}^{\infty}\frac{(4k+1)!!}{(2k+1)!\,2^{2k+1}}\left[(1-\frac{1}{2^{4k+3}})\,\zeta(4k+3)-1\right] \simeq 2.61882\ldots \tag{16}$$

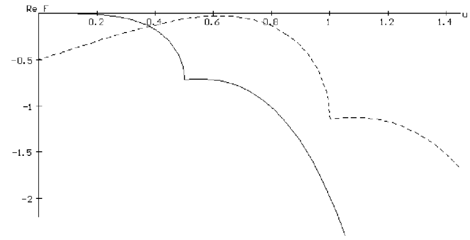
The instant of the phase transition is depicted on the Figure. Thus we have found out the phase transition (presumably of the first order) giving rise to nonzero value of the variable at infinity

$$\sigma^0_\infty = \frac{\pi}{\beta\,g}\,.$$

For SU(3) group the situation is mainly quite similar. It is easy to show that under the same assumption one has [11]

$$\mathcal{U}[s^a,v^a] = \mathcal{U}[s_3,v_3] + \sum_{\pm}\mathcal{U}[(s_3 \pm \sqrt{3}\,s_8)/2, (v_3 \pm \sqrt{3}\,v_8)/2]\,. \tag{17}$$

Thus the effective action is invariant under the centre $\mathbf{Z}_3$ of SU(3) acting by shifts $s_8 \to s_8 + \frac{2}{\sqrt{3}}$ and $s_3 \to s_3 + 1, \ \ s_8 \to s_8 + \frac{1}{\sqrt{3}}$. Hence it suffice to con-



sider it only on the fundamental region

Figure:

Re F's form at the instant of the phase transition — solid line: $s = 0.5, \ a = 2$ and for emergence of nonstable minimum — dashed line: $s = 0, \ a = 0.38$.

formed by hexagon in $\{s_3, s_8\}$-plane. After the phase transition below $T_{SU(3)} = \sqrt{3/2}\ T_{SU(2)}$ the two nontrivial deepest minima laying inside this hexagon $\vec{s}_\pm = (\frac{1}{2}, \pm\frac{1}{2\sqrt{3}})$ arise. Finally the third type of minimum available $\vec{s}_0 = (0, \frac{1}{\sqrt{3}})$ emerges at the temperature $\sqrt{2}\, T_{SU(2)}$.

The essence of this phase transition can be clarified by the help of long–range dynamics formalism by G. Morchio and F. Strocchi [7-9]. Roughly speaking, this elaborated algebraic theory considers models in which quantum equations contain delocalized variables commuting with all local ones. They are usually referred to as *variables at infinity* and belong to centre of the observables algebra. In our case one gets $\forall\ A$

$$[\,\sigma^a_\infty(\hat{\mathbf{x}})\ ,\ A\,] = -w\text{-}\lim_{R\to\infty}\int_0^\infty dy\, f_R^{'}(y)\int_0^\infty \frac{z^2\,dz}{\max(y,z)}\,[\,\nabla\mathbf{E}^a(z\hat{\mathbf{x}})\ ,\ A\,] = 0\,. \tag{18}$$

The last is true by virtue of finite integration region and the limit is understood in the weak sense. Such quantities have definite c-number values $\pi(\sigma_\infty)$ in any primar representation. Quantum dynamics is implementable by means of any Hamiltonian only in irreducible (vacuum) representations for which the corresponding values of variables at infinity satisfy certain triviality condition. The explicit dependence of YM evolution on this values is as follows

$$\dot{A}^a_{\perp\ k} = E^a_{\perp\ k} - P_{kl}\nabla_l(\,\sigma - \pi(\sigma_\infty)\,)^a\,,\qquad P_{kl}\equiv\delta_{kl} - \hat{x}_k\,\hat{x}_l\,, \tag{19}$$

$$\dot{E}^a_{\perp\ k} = gt^{abc}\,E^b_{\perp\ k}\,(\,\sigma - \pi(\sigma_\infty)\,)^c - P_{kl}\nabla_i F^a_{il}\,. \tag{20}$$

This dynamics is generated by the Hamiltonian with cut–off

$$\mathcal{H}_R = H^{YM}_R + \Phi_R\,(\pi(\sigma_\infty) - \sigma_\infty)\ ,\qquad \Phi_R(\varsigma) = \int_{|\mathbf{x}|\le R''} d\mathbf{x}\,\nabla\mathbf{E}^a_\perp(\mathbf{x})\,\varsigma(\hat{\mathbf{x}})^a\,, \tag{21}$$

here the standard Yang–Mills Hamiltonian is given by

$$H^{YM}_R = \frac{1}{2}\int d\mathbf{x}\, f_R(x)\,[\ (\mathbf{E}^a)^2 + (E^a_\parallel)^2 + \frac{1}{2}(F^a_{ij})^2\ ]\,, \tag{22}$$

and cut–off function is (4). The latter Hamiltonian is invariant under the group generated by the cut–off charges $\Phi_R(\varsigma)$ with arbitrary functions of angles $\varsigma(\hat{\mathbf{x}})$

$$[H^{YM}_R\ ,\ \Phi_R(\varsigma)] = 0\,,\qquad [\Phi_R(\varsigma')\,,\,\Phi_R(\varsigma'')] = -i\,\Phi_R(\,[\varsigma'\,,\,\varsigma'']\,)\,. \tag{23}$$

These transformations enlarge every representation of the kind via including elements of nontrivial centre generated by variables at infinity. They form the group of *gauge transformations at infinity* $G_\infty = G/G_{pr}$, where G is the whole gauge group and G_{pr} the subgroup of proper (small) gauge transformations (for $g\in G_{pr}$ $g(\mathbf{x}) \stackrel{|\mathbf{x}|\to\infty}{\longrightarrow} 1$). It is somewhat surprising that the Hamiltonian (21) coincides with those obtained by applying the Regge–Teitelboim ideology [13] from the initial YM Hamiltonian under the behavior $A_0 \sim const$ at spatial infinity. This is clear from the fact that the extra term

$$\Phi_R(\varsigma) = \int d\hat{\mathbf{x}}\, \varsigma^a(\hat{\mathbf{x}})\; R^2\, E^a_{\|}(R\hat{\mathbf{x}}) \tag{24}$$

was a boundary one before reduction.

The (unique) extension of time evolution to the enlarged algebra gives rise to nontrivial *dynamics at infinity.* In the Hartree–Fock approximation this dynamics represents colour rotations

$$\dot{\sigma}^a_\infty(\hat{\mathbf{x}}) = -g t^{abc}\, \sigma^b_\infty(\hat{\mathbf{x}})\, \pi(\sigma_\infty)^c\,. \tag{25}$$

The triviality conditions may be written as

$$Z[\beta, \pi(\sigma_\infty)] = Z[\beta, 0] \tag{26}$$

Then the $\pi(\sigma_\infty)$'s corresponding to vacuum representations are

$$\pi(\sigma_\infty)^a \equiv i\, \sigma^a_n, \quad |\, \sigma_n \,| = \frac{2\pi\, n}{\beta\, g}, \;\; n \in \mathbf{Z}^+\,. \tag{27}$$

They are purely imaginary as well as the equilibrium value of σ_∞ earlier obtained. In the SU(3) case similar formula is obtained from that $\exp(i\, \beta\, \Phi_R(\sigma_n))$ is the element of the centre of this group. The dynamics at infinity depends on σ_n and its Hamiltonian is defined via Kirillov canonical structure [14].

Then the carrier space stable under both time evolution and symmetry transformations is a direct integral of primar representation spaces.

Since there is a direct relation between time evolution of a system and its equilibrium state [15], i. e. via the KMS condition $\omega_\beta(\, A\, \alpha^t[B]\,)\Big|_{t=i\,\beta} = \omega_\beta(\, B\, A\,)\,,\ \forall\ A, B$ on the equilibrium state ω_β at inverse temperature β with respect to dynamics α^t, we have to take dynamics at infinity into account while constructing the Gibbs state.

The equilibrium state corresponding to naive partition function in a primar representation with a fixed value of imaginary σ_∞ constructed as the Gauss state with the Hamiltonian (21)

$$Z_R[\beta, \sigma_\infty - \sigma_n] = \mathrm{Tr}\, e^{-\beta\, [H^{YM}_R - i\, \Phi_R(\sigma_\infty - \sigma_n)]} \tag{28}$$

does not satisfy desired KMS condition under $R \to \infty$.

Namely, the correct averaging procedure should include integration over variables at infinity with Gibbs factor $e^{-\beta\, h}$, where $h(\sigma_\infty)$ is the Hamiltonian of dynamics at infinity, as well as summation over different vacuum sectors. Proceeding in this way we get eventually the equilibrium state satisfying the KMS condition and invariant under symmetry transformations.

It may be shown that summation over vacuum representations (27) obliterates specific features of dynamics at infinity, and the correct partition function takes the form

$$Z = \int d\sigma_\infty \, Z[\beta, \sigma_\infty] \,. \tag{29}$$

We would like to stress that this representation for Z is a direct consequence of **nontrivial** dynamics of variables at infinity in contrast to the standard picture of spontaneous symmetry breaking. In (29), $d\sigma_\infty \equiv \prod_{\hat{\mathbf{x}}} d\sigma_\infty(\hat{\mathbf{x}})$ is the Haar measure of G_∞ group, and integration is to be performed over a convex region including the equilibrium value of σ_∞. This is the whole group range below critical temperature as follows from our previous evaluation. Taking (28) into account we see that

$$Z = \begin{cases} \mathrm{Tr}\, e^{-\beta\, H_{YM}}\,, & T > T_c \\ \mathrm{Tr}\,(e^{-\beta\, H_{YM}}\, P_s), & T < T_c\,, \end{cases} \tag{30}$$

where P_s is the *local singlet projector at infinity* by virtue of the well–known Peter–Weyl theorem [16]. The same projector P_s appears inside any localized variables' correlation functions below T_c. Its emergence prevents localized colour objects from propagating to spatial infinity leading to physically acceptable picture of colour confinement.

The chief issue of our investigations is thus the following. On quantum level the Yang–Mills theory accepts natural extension via incorporating extra classical degrees of freedom. The latter are conjugate to generators of gauge transformations at infinity, in the sense that they contribute to the partition function as in (28). At temperatures below T_c the equilibrium values of these quantities are nonzero. This fact entails an abrupt change of the equilibrium state structure at T_c providing colour confinement.

It should be stressed that these statements are in fact gauge independent. The specific rôle played by the Fock–Schwinger gauge is that the necessity of incorporating variables at infinity manifests itself most clearly. Having established this fact we are able to use any gauge for practical calculations. Namely, it is easy to show in general case

$$Z[\beta] = \int_{G_\infty} d\zeta \int \mathcal{D}A \,\mathrm{Det}\left[\frac{\delta\,\chi(A^\omega)}{\delta\,\omega}\right]_{\omega=0} \delta[\chi(A)] \exp\left\{-W_{_{YM}}[A_0 + (g\beta)^{-1}\zeta, \mathbf{A}]\right\} \tag{31}$$

where $\chi[A] = 0$ is the gauge condition. In (31) the path integral over A is evidently the partition function in presence of the external field $A_0^{ext} = (g\beta)^{-1}\zeta(\hat{\mathbf{x}})$. It is worth to note also that integration over ζ restores gauge- and Poincaré- invariance of Z. As for the $T = 0$ limit, it is permitted in formulae like (31) only *after* the ζ-integration.

The formula (31) provides link with more conventional approaches. Recently the problem of A_0-condensation has attracted certain interest both in the field theory and on lattice [17, 18]. It was also realized that it is closely related with confinement phase transition. However the obtained results are not very coherent in different approaches. It is interesting that the first order phase transition giving rise exactly the same value of A_0 as predicted here can be seen at g^2 large enough in [17]. However this fact wasn't especially appreciated in that work. Moreover the effective action at this extremal point ceases its gauge dependence.

We entertain a hope that the use of (31) may facilitate the calculation of condensates, correlation functions etc., and the analysis of renormalization procedure, that is in any case indispensable for complete substantiation of our approach. However these points are not clear yet.

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